A Rate-Dependent Damage Model for Brittle Materials under Dynamic Loading

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We have recently developed a rate-dependent continuum damage model (Dominant Crack Algorithm, or, DCA) for brittle materials under dynamic loading [1]. The model is derived from the responses of an ensemble of penny-shaped microcracks which are assumed to be randomly distributed within a statistically homogenous volume of a brittle material. It is assumed in the current model that the distribution of microcracks remains isotropic during loading. The main features of the model are: a) the damage tensor is derived from opening and shear of an ensemble of penny-shaped cracks with various orientations and sizes; b) the evolution of damage (through crack growth) is based on the energy-release rate for the dominant crack (having the most unstable orientation); and c) the damage surface, outside which material accumulates additional damage, is found by applying the generalized Griffith instability criterion to the dominant crack. The use of the energy-release rate for damage evolution provides a physical means to introduce rate effects in our damage model. See [1] for details of the model.

To illustrate the features of the model, several standard load paths (isotropic, uniaxial strain, uniaxial stress, and pure-shear) have been simulated with the stand-alone driver program, which provides the model with a strain history. Figure 1 shows: a) the stress response, and b) the evolution of the average crack radius (damage), to an isotropic ($\varepsilon = \varepsilon_{11}\mathbf{i}$, $\sigma = -P\mathbf{i}$), cyclic loading, with a strain rate $\varepsilon_{11} = 10^5/\mathrm{s}$. The model material is a SiC ceramic [1]. The material is initially stress-free (point **A**) and is first loaded up to a tensile strain of 0.01 (**C**), then unloaded back to zero strain (**A**) and reverse loaded (compression) to a strain of $\dot{\varepsilon}_{11} = -0.0005$ (**D**), and finally reloaded to a tensile strain of 0.0195 (**E**). The initial loading path (**A-A′-B-C**) begins with an elastic response **A-A′**, with the slightly damaged modulus corresponding to the initial crack size $\bar{c}_0 = 14\mu m$. When the stress reaches the initial damage surface (**A′**), initiating crack growth, this causes the damage surface to contract with further straining. Though the size of the damage surface starts to decrease immediately due to crack growth, the stress level in the material still increases with strain until a peak value (**B**) is reached. This is because the rate of damage accumulation, which is proportional to the square of the crack size, is small when the crack size is small, and the inelastic strain rate due to the crack growth is too small to influence the total strain rate significantly. Consequently, the response remains "strain-hardening" (**A′-B**). Because $\dot{\varepsilon}_c^{gr}$ (the inelastic strain rate due to crack growth) increases with crack size and distance from the stress state to the damage surface, for a given total strain rate $\dot{\varepsilon}$, the inelastic strain rate $\dot{\varepsilon}_c^{gr}$ eventually approaches the total strain rate and the material response changes from hardening to softening (**B-C**).

The unloading path (**C-C′-A**) begins at **C** and, because the stress state is outside the damage surface, crack growth continues until the stress unloads enough to reach the surface (point **C′**). From **C′**, the material unloads elastically (with the damaged modulus) back to the origin (**A**), where both the matrix strain and crack strain are zero, and all the cracks are completely closed. The segment **A-D** corresponds to reverse loading (hydrostatic compression) of the damaged material with the crack size attained at **C′**($\bar{c}_1 = \bar{c}' \approx 10\bar{c}_0$). Because the cracks remain closed under compression, damage accumulated in the material is deactivated (cracks of size $\bar{c}_1$ are still present). Consequently, the material assumes the original (undamaged) stiffness. The reloading path (**D-A-C′-E**) starts from **D** and continues elastically with the undamaged stiffness back to the origin (**A**). On further loading, the cracks (with the increased size $\bar{c}_1$) open under tension and the damage, which has been accumulated at **C′**, becomes active again. Consequently, the reloading path follows the segment **A-C′**. The path intersects the damage surface at point **C′**, and the crack size again increases along the path (**C′-E**). The stress state is outside the damage surface due to rate effects.

It is shown in Fig. 1(b) that the cracks are initially stable when the stress level is low (**A-A′**), become unstable at **A′**, and grow rapidly at first due to the high values of energy release rate, then slowly as the stress level drops. On unloading, the cracks continue to grow slightly (**C-C′**), and then arrest and remain stable (the stress state is inside the damage surface). During reloading, the cracks remain stable (**D-C′**) until the stress reaches the damage surface again at point **C′**. During the rest of the reloading path (**C′- E′**), cracks continue to grow.

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[1] Q.H. Zuo, et al., "A Rate-Dependent Damage Model for Brittle Materials Based on the Dominant Crack," *Int. J. Solids Struct.*, in press, pp. **1–31** (2005) (available online at the journal's website.)

Fig. 1.
The predicted response under isotropic, cyclic loading: a) The pressure-strain response; and b) Evolution of the crack size as a function of the strain.

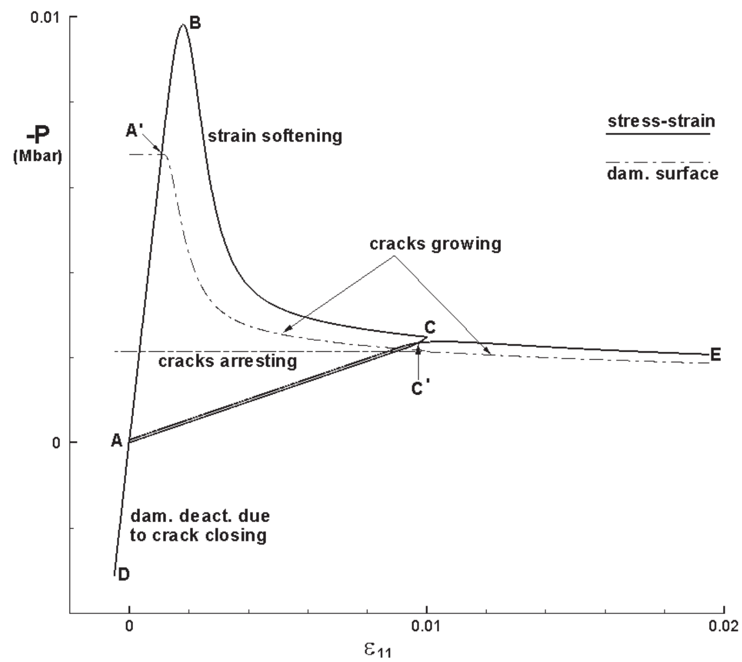

Fig. 1(a)

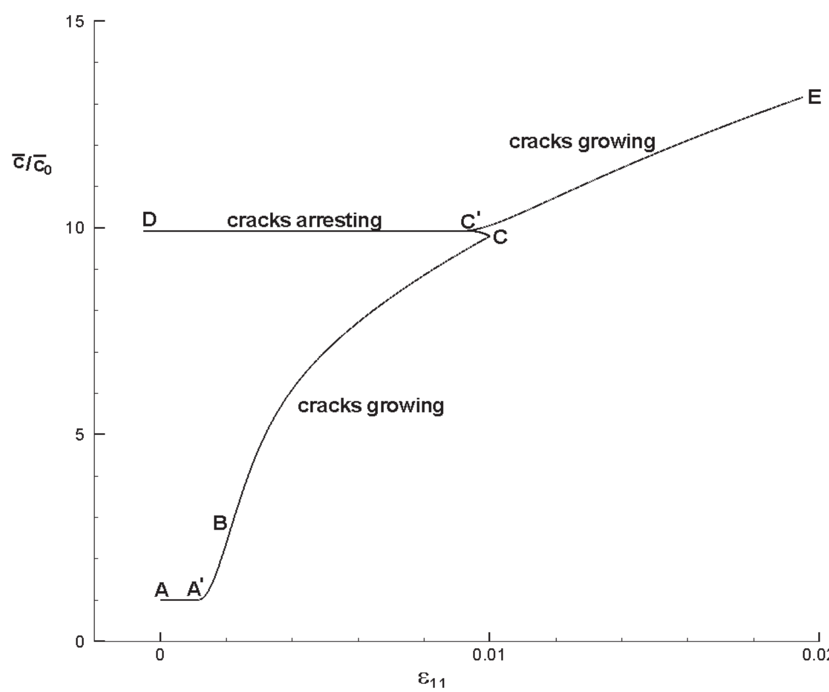

Fig. 1(b)